

Practice 6A



Basic

1. The cumulative frequency table shows the distribution of time taken by a group of students to complete a project.

Time (in minutes)	Number of students
20 or less	0
30 or less	2
40 or less	8
50 or less	15
60 or less	24
70 or less	28
80 or less	30

From the table,

- how many students participated,
- how many percent of students took 60 minutes or less to complete the project,
- how many students took more than 60 minutes to complete the project?

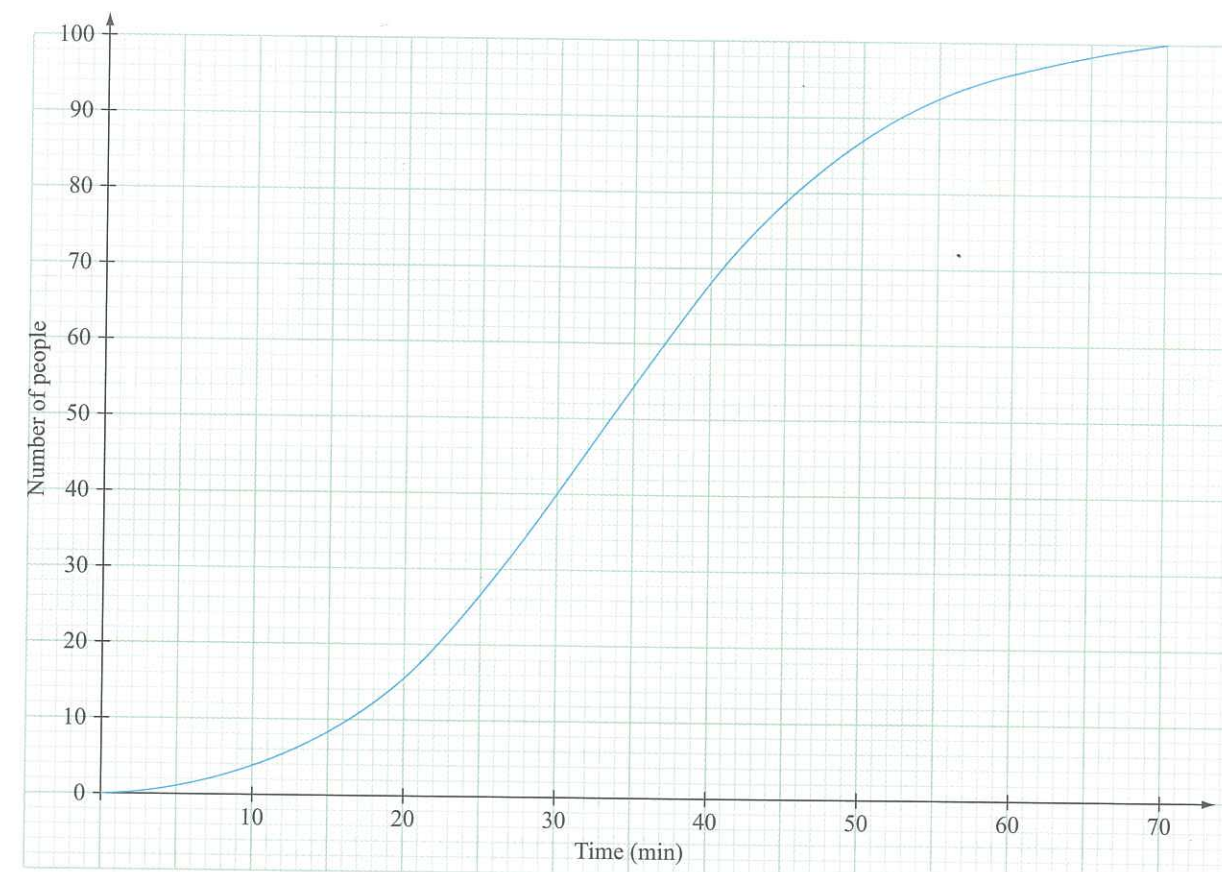
2. A group of swimmers measured the length of time they could hold their breath in water. The results were shown in the cumulative frequency table below.

Time (in seconds)	Number of swimmers
20 or less	0
25 or less	3
30 or less	18
35 or less	40
40 or less	50
45 or less	58
50 or less	60

Use the table to find the number of students who

- can hold their breath for more than 40 seconds,
- can hold their breath for less than or equal to 30 seconds.

3. The following cumulative frequency curve summarises the survey of 100 people on time spent in travelling to work.



From the graph, estimate

- the number of people who took less than 35 minutes to go to work,
- the number of people who took 45 minutes or more to go to work.



Advanced

4. A survey was carried out to find out the amount of money each family spent on food every month. The findings were summarised in the table given.

Amount less than	Number of families
\$220	10
\$260	28
\$300	62
\$340	88
\$380	96
\$420	100

- How many families spent at least \$340 on food each month?
- How many families spent at least \$220 but less than \$300 on food each month?
- What is the percentage of the families who spent at least \$220 but less than \$380 on food each month?

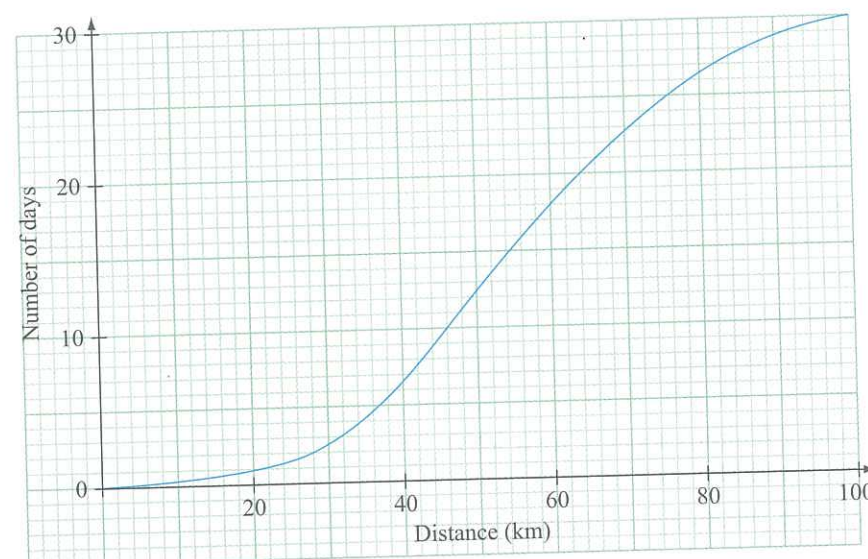
5. The following table shows the results of a survey on the number of books read by some students in a month.

Number of books	Number of students
$0 \leq x < 10$	10
$10 \leq x < 20$	18
$20 \leq x < 30$	20
$30 \leq x < 40$	9
$40 \leq x < 50$	3

The above table can be represented as shown below.

Number of books (x)	$x < 10$	$x < 20$	$x < 30$	$x < 40$	$x < 50$
Number of students	10	A	48	57	B

- (a) Find the values of A and B.
 (b) How many students read fewer than 10 books in a month?
 (c) How many students read 40 books or more in a month?
 (d) If a student who reads 30 books or more can receive a good reader badge,
 (i) how many students will receive this badge,
 (ii) what percentage of the total number of students will receive the badge?
6. The following cumulative frequency curve shows the distance travelled in km by Mr Kong in the month of June.



- (a) How many days did he travel less than 50 km?
 (b) How many days did he travel 70 km or more?
 (c) He travelled 79 km and above for x days. What is x ?
 (d) He travelled not more than 60 km for y days. What is y ?

6.2 Quartiles, Percentiles and Box-and-Whisker Plots

At times we wonder how spread out a set of data is. To put it in another way, do the values of data vary greatly from one another? In this section, we shall learn how to measure variability of data, i.e. the extent to which the data vary from one another.

Range

Suppose there are 16 adults, both young and old, attending a cooking class. The ages of the participants are as follows:

50	62	21	38	50	48	44	24
43	27	34	20	34	25	32	30

Let us list their ages from the youngest to the oldest:

20, 21, 24, 25, 27, 30, 32, 34, 34, 38, 43, 44, 48, 50, 50, 62

To gauge how wide the age group is, we find the difference in age between the oldest and the youngest. In this case, the difference = $62 - 20 = 42$. This value is called the **range**.

In general, given a set of data,

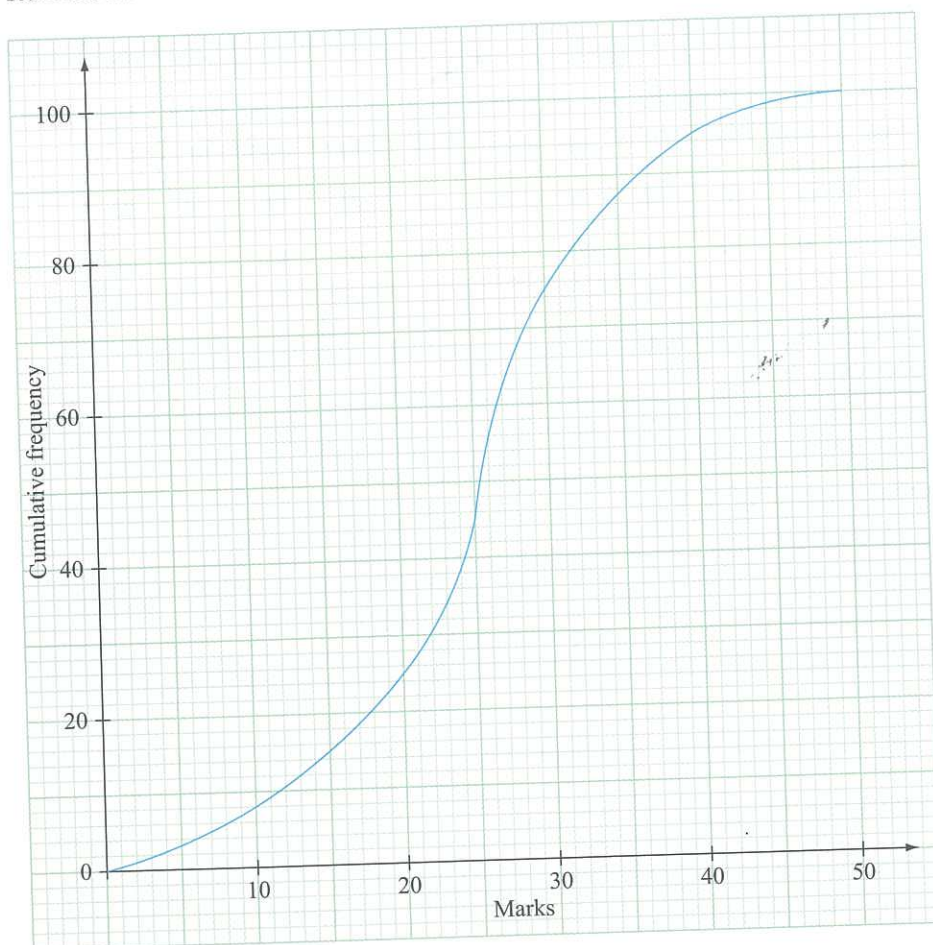
range = the highest value – the lowest value

Practice 6B



Basic

- The following graph shows the cumulative frequency curve of the marks obtained by a group of students in an examination.

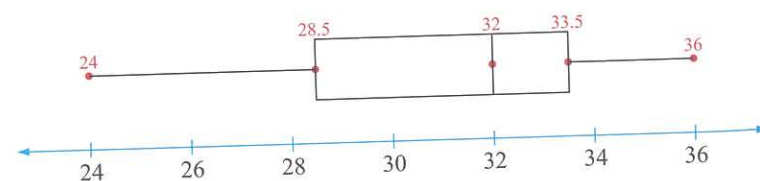


- Find (a) the lower quartile, (b) the median,
(c) the upper quartile, (d) the interquartile range.

- From the box-and-whisker plot given, read off

- the lower quartile,
- the median,
- the upper quartile,
- the minimum value,
- the maximum value.

Hence find the interquartile range.



Advanced

- The time taken by 80 students to solve a problem in a competition was summarised in the cumulative frequency curve given.

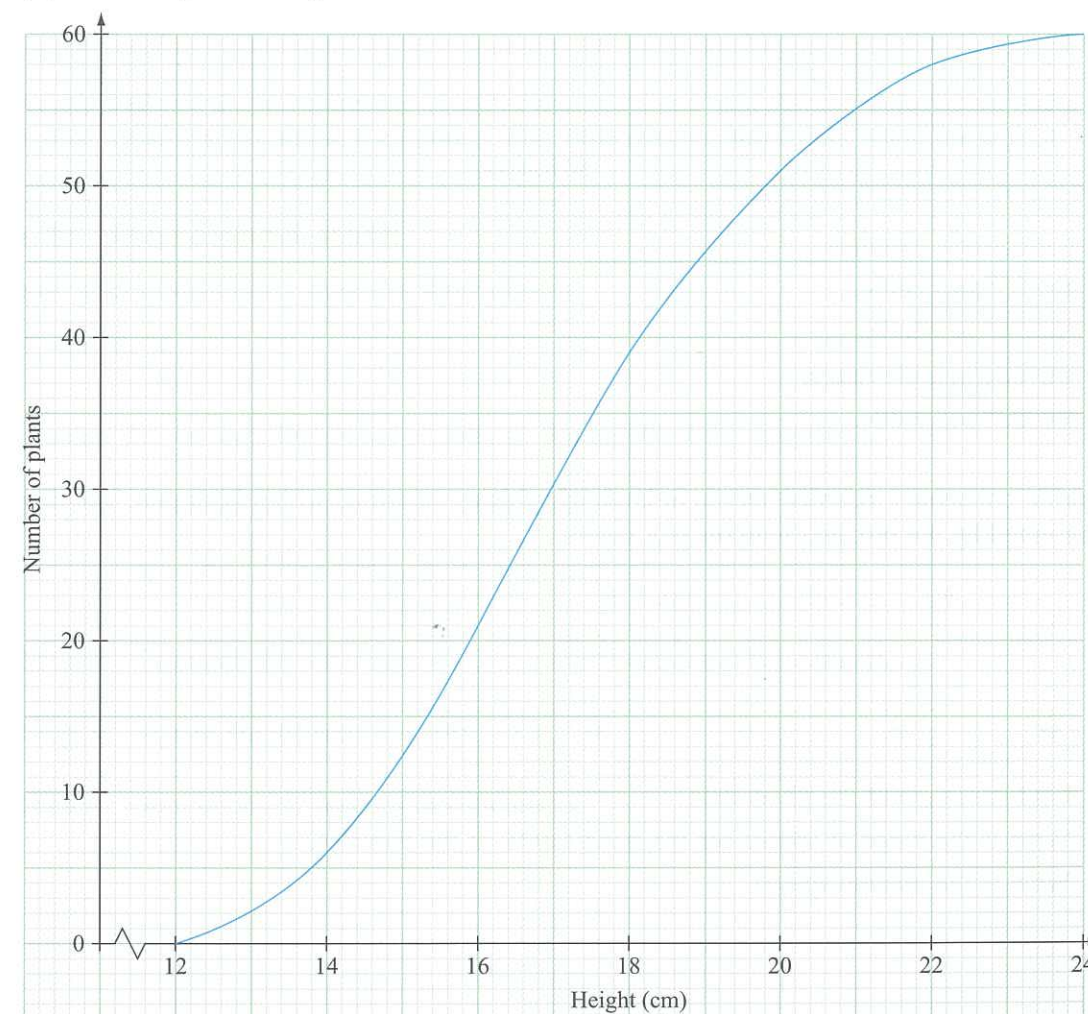
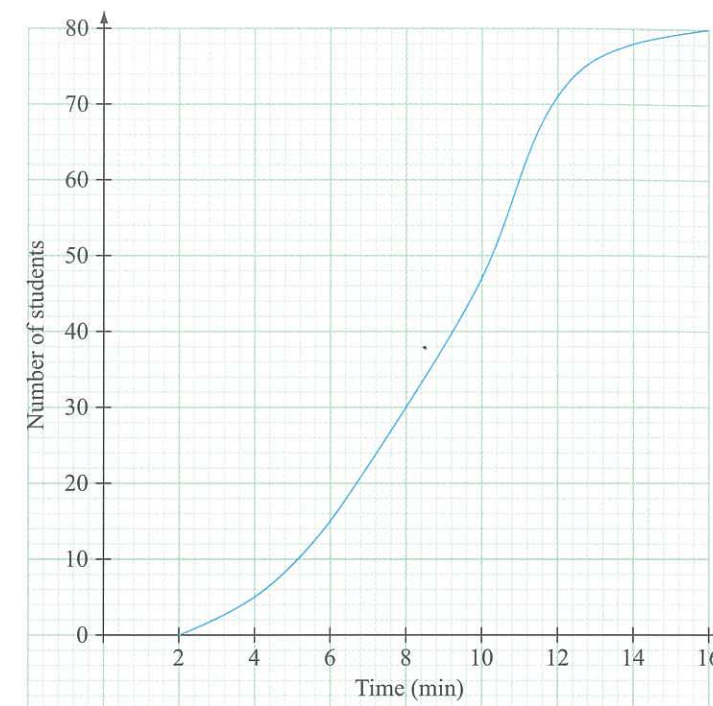
From the graph, estimate

- the 30th percentile,
- the 70th percentile.

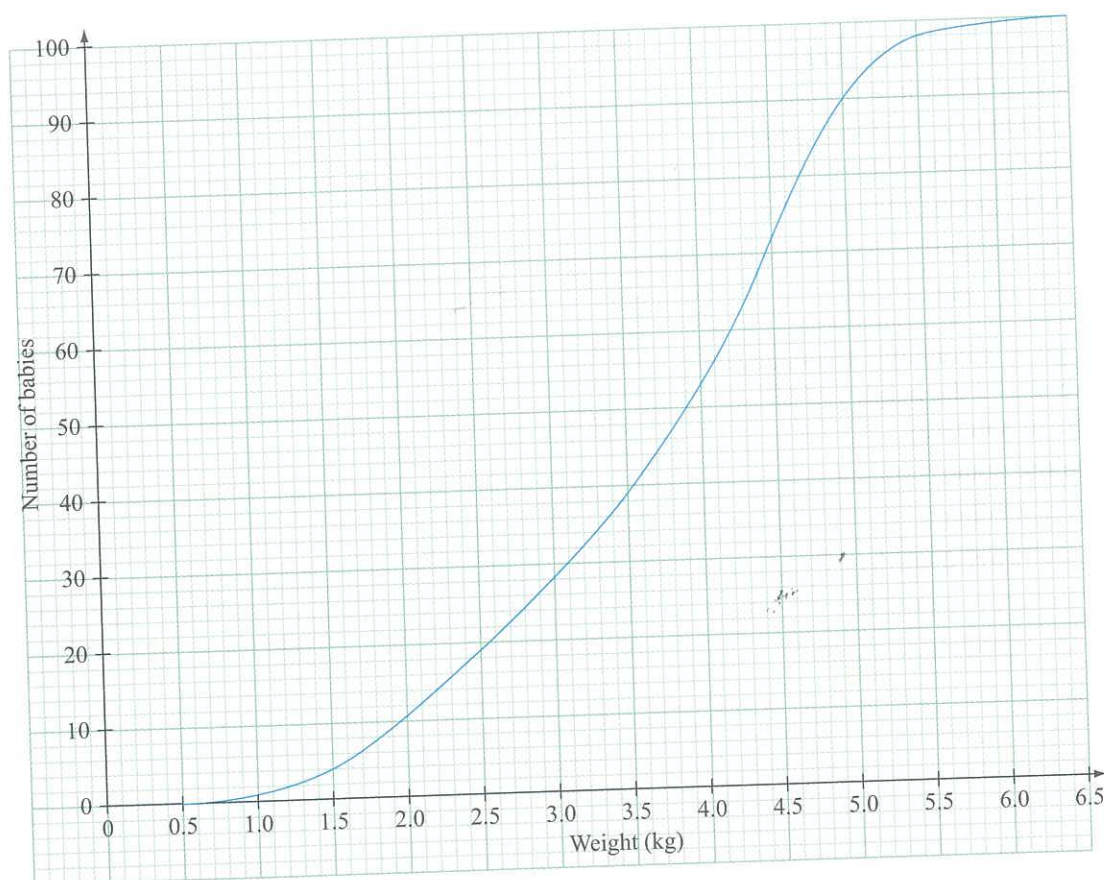
- The given cumulative frequency curve shows the heights of 60 plants in a nursery.

From the graph, estimate

- the median,
- the lower quartile,
- the upper quartile,
- the interquartile range.

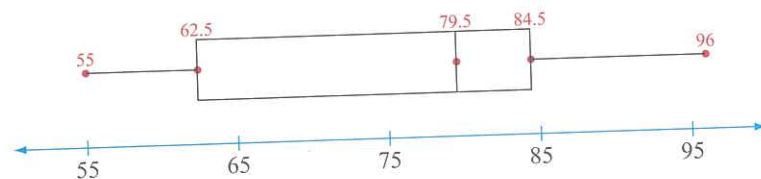


5. In a maternity ward, the weights of 100 babies were summarised as follows.



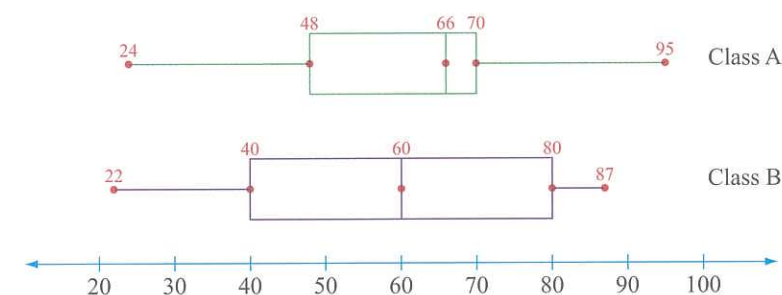
From the graph, estimate

- the 25th percentile,
 - the 75th percentile,
 - the interquartile range,
 - the 80th percentile.
6. A traffic policeman recorded speeds in km/h of vehicles on a road with a laser gun. The data are represented by the box-and-whisker plot below.



- Find the range of the given data.
- What is the range of speeds for the middle 50% of the vehicles?
- What is the median speed?

7. Two classes took a mathematics test and the results were summarised in the box-and-whisker plot given.



- Which class has data more spread out?
- Comment on the performance of two classes.

6.3 Variance and Standard Deviation

Unlike the statistics that measure the spread of data, **variance** and **standard deviation** make use of all the data in a distribution. These 2 measures show how data are spread out from the mean.

Case 1: Ungrouped data

Suppose a set of data has 5 values as shown below:

2, 4, 6, 8, 10

Maths Vitamins
 \bar{x} is pronounced as 'x bar'.

To calculate the variance for the set of data, go through the following steps.

Step 1: Calculate the mean of the set of data, say \bar{x} .

$$\bar{x} = \frac{2 + 4 + 6 + 8 + 10}{5} = 6$$


Step 2: Calculate the difference $x - \bar{x}$ for every value x in the data set.

Step 3: Square the difference, i.e. $(x - \bar{x})^2$.























Step 4: Sum up all the squares of differences.

Now the calculator is ready for data input.



Data input

We enter the data values using the data entry key .

Suppose we want to enter 8 values: 24, 30, 27, 27, 48, 39, 18, 22. We press the following keys.





Note that

- when a value is entered, it must be separated from the next value by pressing the  key.
- the  key is pressed twice to indicate that the number 27 has two occurrences.





Now all the 8 values have been stored in the memory for use.

Statistical calculation

Using the values stored in memory, we can find the sum and average of data values, the variance and standard deviation between each data value.

Statistical calculation	Key
sum of data values	
average of data values	
variance between each data value	
standard deviation between each data value	

By pressing the keys in your calculator, you will obtain the following results.

Statistical calculation	Result
	7567
	235
	29.375
	9.11

Check your own calculator for the location of the keys.



Go to Workbook Exercise 3, page 116

Practice 6C



Basic

- Find the variance and standard deviation of each of the following set of data, giving your answers correct to 2 decimal places where necessary.
 - 2, 6, 4, 4, 7, 3, 4, 6, 8, 5
 - 50, 78, 90, 55, 68, 81, 66, 72, 56, 64, 46, 45, 45, 78, 67, 60
 - 277, 380, 382, 192, 300, 196, 274, 494, 396, 230
 - 4, 5.6, 7.8, 9, 10.2, 8, 12.7, 13.4, 9.1, 6, 8.8, 10

- The weights of eggs were recorded as follows.
68 g, 54 g, 60 g, 61 g, 51 g, 56 g, 69 g and 53 g
Find the variance and standard deviation of the weights of eggs.

- The temperatures (in degree Celsius) of the past two weeks were tabulated below.

31.5	28	29	27.2	24	25.5	24.6
30	30	26	25	29	32	28.8

Find

- the mean temperature,
 - the standard deviation of the temperatures.
- A technician measured the expansion of metal bars (in mm) and captured the following measurements.

Expansion in mm (x)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Number of bars (f)	1	5	40	50	36	30	25	10	3

- (a) Fill in the following table.

x	f	x^2	fx	fx^2

- (b) Calculate the mean measurement.
 (c) Find the variance and standard deviation of the measurement distribution. Give your answers correct to 3 decimal places.



Advanced

5. 88 children participated in a game where the count of hits at the target out of 50 trials is kept for each child and is tabulated in the following grouped frequency table.

Number of hits (x)	$0 \leq x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$
Number of children (f)	6	16	24	25	17

For the grouped distribution, find

- (a) the mean,
 (b) the variance, and
 (c) the standard deviation.
 Give your answers correct to 2 decimal places.

6. The life span (in hours) of a certain type of electronic gadget was recorded in the following grouped frequency table.

Life span (in h hours)	Number of gadgets
$300 \leq h < 400$	13
$400 \leq h < 500$	25
$500 \leq h < 600$	66
$600 \leq h < 700$	58
$700 \leq h < 800$	38

Find the standard deviation of the life span of the gadgets, correct to 2 decimal places.



Applications

7. For the past 12 months, the monthly mathematics test results of Evelyn and Jasmine were recorded in the table below.

Evelyn	80	70	50	68	70	85	60	75	50	66	58	60
Jasmine	80	84	75	78	80	81	78	70	84	74	85	79

- (a) Calculate the means and standard deviations of the test results of Evelyn and Jasmine.
 (b) Compare the means and standard deviations of the two girls.
 (c) What could you comment on their performance in mathematics for the past 12 months?
 Give your answers correct to 2 decimal places where applicable.
8. In an online game, Ken and Roger are supposed to shoot at a moving target. Each of them is given 10 trials. The response time (in seconds) that each of them takes to hit at the target is kept in the following table.

Ken	10	10	12	15	10	9	8	14	9	13
Roger	16	7	20	10	6	8	14	14	8	7

- (a) Calculate the means and standard deviations of the response time of Ken and Roger. Give your answers correct to 2 decimal places where applicable.
 (b) Compare the means and standard deviations of the two boys.
 (c) What could you comment on their performance in the online game?

Test Yourself

- Determine whether the following statements are true.
 (a) The 50th percentile is the same as the mean.
 (b) The 3rd quartile is the same as the 75th percentile.
 (c) 50% of a data set lies between the median and the 3rd quartile.
 (d) 1 standard deviation is the same as the 1st quartile.

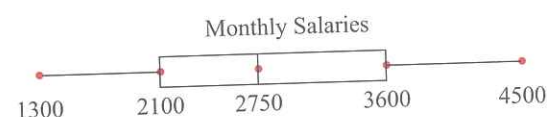
(True/False)

(True/False)

(True/False)

(True/False)

- The box-and-whisker plot on the right shows the starting monthly salaries (in dollars) for new executives in a company.



- What is the range of the starting salaries?
- What is the median salary?
- What is the interquartile range?

- Standard deviation measures how data are spread around
 (a) mean. (b) median. (c) mode.

- Among the measures of variability, the one that uses all the values in a data set is
 (a) range (b) quartiles (c) interquartile range (d) variance

- The history test of two classes A and B has the following statistics.

	Mean score	Standard deviation
Class A	75	4.32
Class B	76	8.40

Circle as many statements below as possible that best describe the performance of the students of the 2 classes.

- It is more difficult to gauge how well a student from class A performed in the test than a student from class B.
- Class A students' performance is relatively uniform.
- Performance of the students from the 2 classes is about the same since the mean scores of the 2 classes are very close.
- It is more likely that the interquartile range of scores in class A is smaller than the interquartile range of scores in class B.

Go to Workbook Revision Exercise 6, page 118

Cumulative Practice 6



Basic

- The scores of a recent mathematics competition were summarised in the cumulative frequency table given.

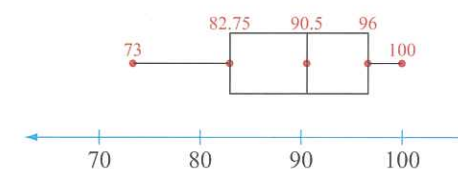
Score less than	Cumulative frequency
40	0
50	8
60	18
70	25
80	34
90	38
100	43
110	47
120	50

- How many participants scored 100 and above?
- What percentage of the participants scored at least 60 but less than 90?

- Given the data set: 6, 4, 3, 2, 4, 4, 4, 7, 2, 6, find

- the range,
- the variance,
- the standard deviation.

- Given the box-and-whisker plot, find



- the range,
- the 1st quartile,
- the maximum value,
- the median,
- the interquartile range.

- In a speed limit zone, a traffic policeman has recorded the speeds (in km/h) of 20 vehicles.

60	74	85	85	60	66	78	73	80	86
80	90	78	90	94	56	50	67	94	82

Find the variance and standard deviation of the speeds of the vehicles. Give your answers correct to 2 decimal places where necessary.

5. Determine the variance and standard deviation of the following values, giving your answers correct to 2 decimal places.

Value (x)	Frequency (f)
37	2
38	4
39	5
40	2
41	2
42	1
43	2
44	2

6. The marks gained by a group of 600 candidates are shown in the frequency table below.

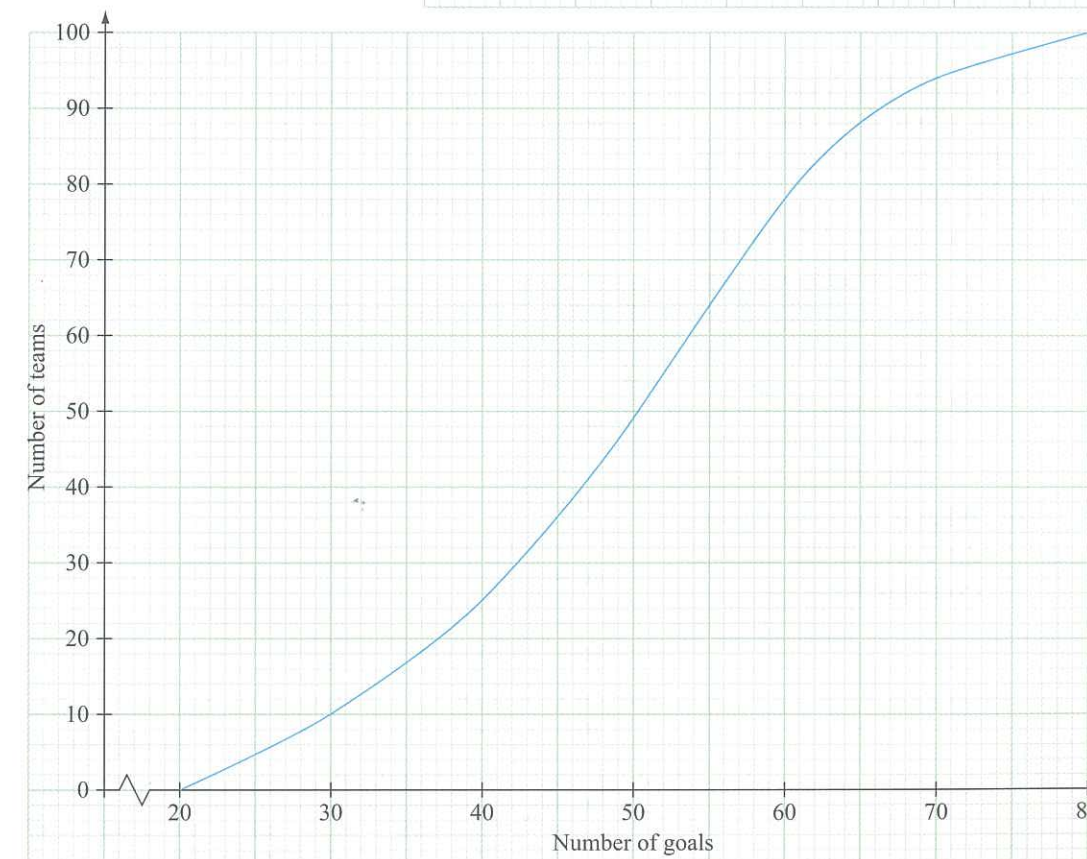
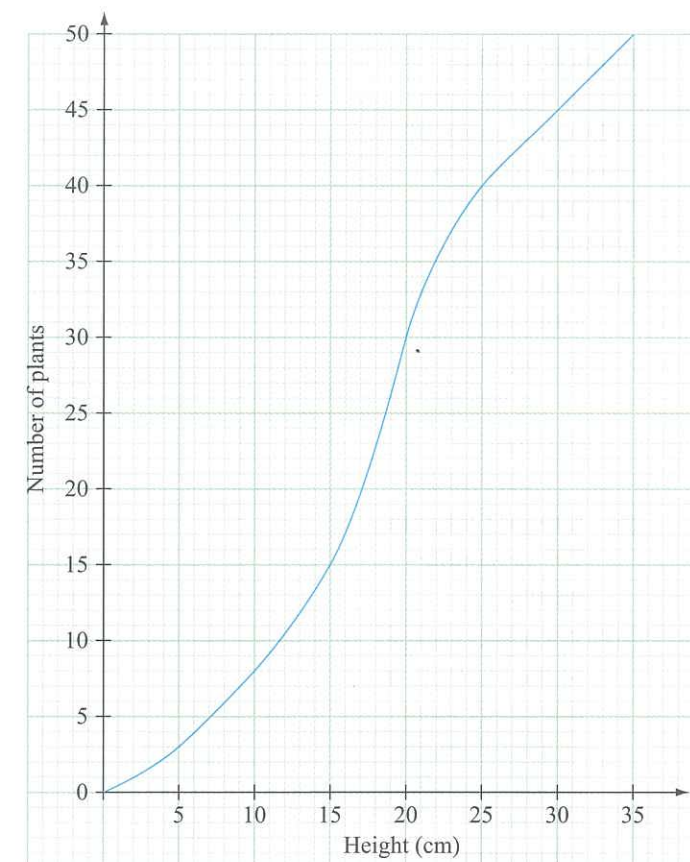
Marks (x)	Frequency
$0 < x \leq 10$	8
$10 < x \leq 20$	24
$20 < x \leq 30$	71
$30 < x \leq 40$	82
$40 < x \leq 50$	107
$50 < x \leq 60$	128
$60 < x \leq 70$	92
$70 < x \leq 80$	61
$80 < x \leq 90$	18
$90 < x \leq 100$	9

- (a) Find the mean mark.
 (b) Find the variance and standard deviation.



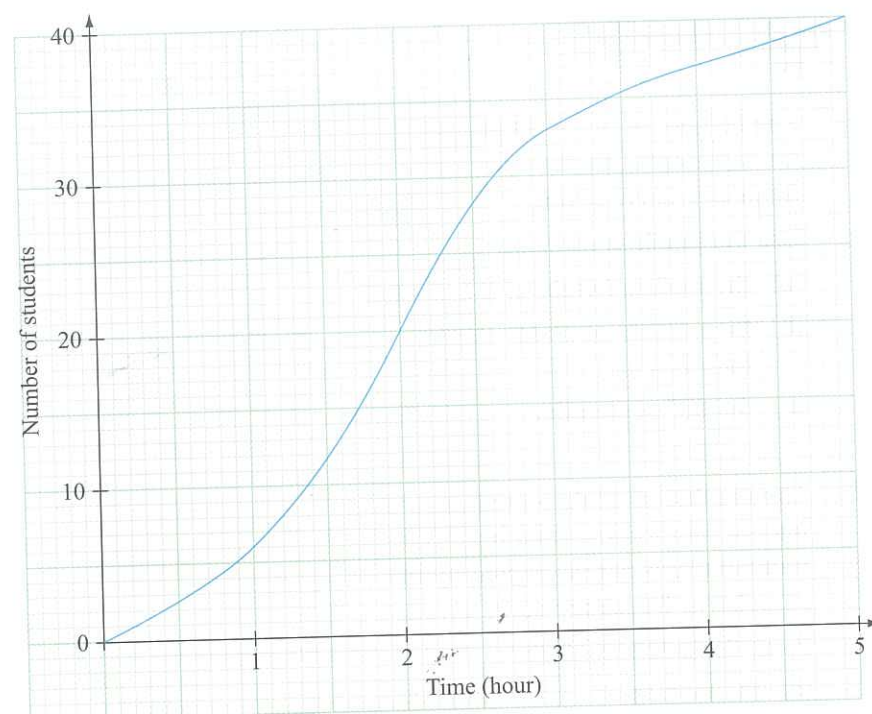
Advanced

7. The heights in cm of 50 plants are represented by the cumulative frequency curve given. Use the graph to estimate
- the median height of the plants,
 - the number of plants shorter than 10 cm,
 - the 60th percentile.
8. The number of goals scored in some inter-school tournaments by 100 football teams is represented by the cumulative frequency curve given. From the graph, estimate
- the median,
 - the 40th percentile,
 - the 1st quartile,
 - the 3rd quartile.



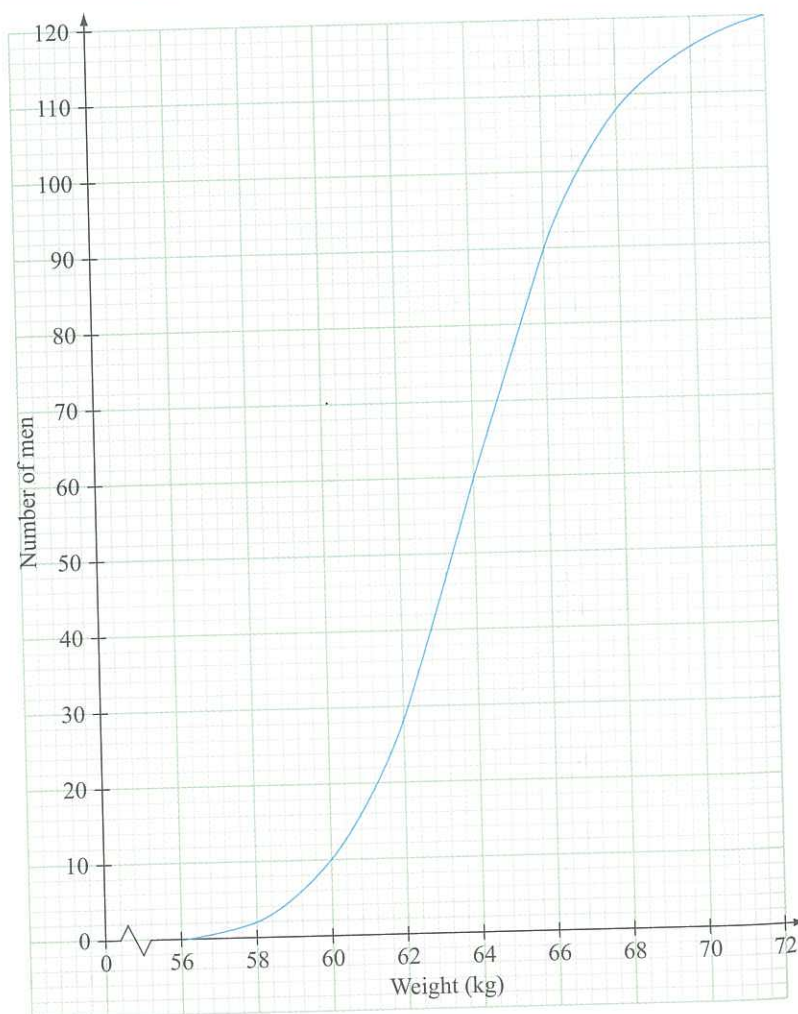
9. The cumulative frequency curve shows the number of hours a group of students spent on watching television programmes per day.

- (a) From the graph, estimate
- the 3rd quartile,
 - the 70th percentile.
- (b) Interpret what the 70th percentile means.
- (c) What percentage of the students in the group spent at least 2 hours per day watching television programme?



10. The cumulative frequency curve given shows the distribution of weights in kg of 120 men.

- (a) Use the graph to estimate
- the median weight,
 - the interquartile range,
 - the number of men who weighed less than 65 kg.
- (b) What is the meaning of the interquartile range in everyday language?



11. A survey was conducted on petrol consumption rates of 100 small cars and the findings were tabulated below.

Consumption rate (in x km/l)	Number of cars (f)
$14 \leq x < 16$	9
$16 \leq x < 18$	13
$18 \leq x < 20$	24
$20 \leq x < 22$	38
$22 \leq x < 24$	16

Calculate

- the mean consumption rate,
- the standard deviation of the consumption rate.

12. Temperatures (in degree Celsius) of 2 cities X and Y were taken over 10 days and were recorded as follows.

City X	30	31	27	32	28	28	26	30	32	30
City Y	25	22	24	30	33	20	18	23	25	29

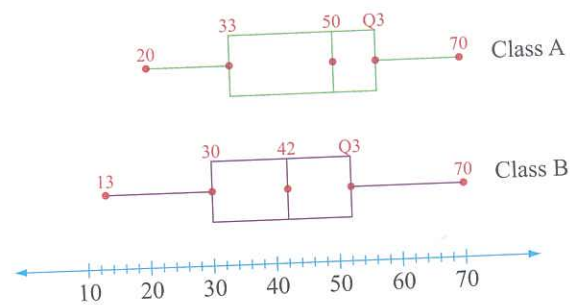
- Find the mean temperatures of the 2 cities.
- Calculate the standard deviations of temperatures of the 2 cities.
- What conclusion can be drawn by comparing the means and standard deviations of temperatures of the 2 cities?

13. The share prices of 2 stocks A and B were captured on an hourly basis on a certain day, as shown below.

Price of stock A (in \$)	Price of stock B (in \$)
1.30	1.05
1.32	1.10
1.29	1.25
1.27	0.80
1.30	1.32
1.24	1.45

- Find the mean and standard deviation of each of these 2 stocks. Give your answers correct to 3 decimal places.
- Based on the means and standard deviations, how do the 2 stocks fluctuate throughout the day?

14. Below are the box-and-whisker plots of the scores of 2 classes in a common test. Both classes have the interquartile range 22.



- Write down the upper quartiles for classes A and B.
- Which class has more varied scores? Give a reason.
- Compare the performance of the 2 classes in the common test.

Chapter Review

- The following concept map summarises the scope of coverage in this chapter.

